# *Basic Concepts of Machine*

**Machine**

Device receiving energy in some available form using it in some particular useful work.

**Lifting machine**

Device capable of lifting heavy load by applying smaller effort.

1. **Load** A load is defined as the force or weight which is to be lifted by the machine. It is denoted by $W$.
2. **Effort** Effort is defined as the force required by a machine to lift a load. It is denoted by $P$.
3. **Mechanical Advantage** Mechanical advantage is the ratio of the load lifted to the effort applied. It is denoted by $MA$ and is given by,

$$MA = \frac{\text{Load Lifted}}{\text{Effort Applied}} = \frac{W}{P}$$

4. **Velocity Ratio** Velocity ratio is the ratio of the distance moved by an effort to the distance moved by the load. It is denoted by *VR* and is given by,

$$VR = \frac{\text{Distance moved by effort}}{\text{Distance moved by load}} = \frac{y}{x}$$

5. **Input** It is the work spent on the machine and is given by the product of the effort and the distance it travels.

$$\therefore \quad \text{Input} = \text{effort} \times \text{distance moved by effort}$$

$$= P \times y$$

6. **Output** It is the useful work obtained from the machine and is given by the product of load lifted and the distance through which it moves.

$$\therefore \quad \text{Output} = \text{load lifted} \times \text{distance moved by load}$$

$$= W \times x$$

7. **Efficiency of a Machine** This is the ratio of output to input. It is denoted by $\eta$ and is given by,

$$\eta = \frac{\text{Output}}{\text{Input}} = \frac{W\,x}{P\,y}$$

$$= \left(\frac{W}{P}\right) \times \frac{1}{(y/x)} = \frac{MA}{VR}$$

Machine efficiency is a fraction less than one and is generally expressed as percentage. No machine exists on this earth whose efficiency is 100%. It means work spent on machine is always larger than the useful work obtained from it.

**8. Ideal Machine** A machine is said to be ideal if its efficiency is 100%. It is not practically possible to get 100% efficiency thus ideal machine is a hypothetical machine.

In the case of ideal machines, friction losses are neglected, so for an ideal machine,

$$\text{Output} = \text{Input}$$

i.e., $$P\,y = W\,x$$

$$\frac{W}{P} = \frac{y}{x}$$

$$MA = VR$$

Thus, for an ideal machine its efficiency ($\eta$) is 100% or $MA = VR$.

**9. Ideal Effort** An ideal effort is defined as the effort required for lifting a load by an ideal machine whose efficiency is 100%. It is denoted by $P_{ideal}$ or $P_i$.

For an ideal machine, $\eta = 1$

$$\therefore \quad \frac{W\,x}{P_i\,y} = 1$$

$$P_i = \frac{W}{(y/x)} = \frac{W}{VR}$$

For actual machine, $$\eta = \frac{MA}{VR} = \frac{W}{P} \times \frac{1}{VR}$$

$$P_{actual} = \frac{W}{\eta} \times \frac{1}{VR}$$

Thus, the extra effort has to be applied to lift the load due to the presence of friction. So, effort wasted in friction can be given as,

$$P_{friction} = P_{actual} - P_{ideal}$$

$$= \frac{W}{\eta} \times \frac{1}{VR} - \frac{W}{VR}$$

$$= \frac{W}{VR} \times \left(\frac{1}{\eta} - 1\right)$$

Equation (5.8)

**10. Ideal Load** An ideal load is defined as the load that can be lifted by an ideal machine by applying an effort. It is denoted by $W_{ideal}$ or $W_i$.

Since, for an ideal machine

$$\eta = 1$$

$$\therefore \quad \frac{W_i x}{Py} = 1$$

$$W_i = P \times (y/x) = P \times VR$$

For actual machine, $\eta = \frac{MA}{VR} = \frac{W}{P} \times \frac{1}{VR}$

$$\therefore \quad W_{actual} = \eta \times P \times VR$$

Thus, smaller load will be lifted due to the presence of friction. So, the decrease in load lifted due to the presence of friction can be given as,

$$W_{friction} = W_{ideal} - W_{actual}$$

$$= P \times VR - \eta \times P \times VR$$

$$= P \times VR \times (1 - \eta)$$

*Reversible and Irreversible Machine*

## 5.3 REVERSIBLE AND IRREVERSIBLE MACHINES

On removal of the effort completely if the load falls down and the machine works in reverse direction, such a machine is known as reversible machine, for example, rope and pulley arrangement. On the other hand, if the load does not fall down on removal of the effort and the machine does not work in reverse direction such a machine is known as irreversible machine, for example, a screws jack. The irreversible machine is also known as self-locking machine.

### Condition for Irreversibility of Machine

Let an effort $P$ be applied through a distance $y$ to lift a load $W$ through a distance $x$, then

$$\text{Input} = P \times y$$

and $$\text{Output} = W \times x$$

Work lost in friction or frictional work

$$= \text{Input} - \text{Output} = Py - Wx$$

In the case of irreversible machine, the load does not fall back on removal of the effort, in other words, frictional work is greater than the output.

$$Py - Wx > Wx$$

or $$Py > 2Wx$$

or $$\frac{1}{2} > \frac{Wx}{Py}$$

or $$\frac{Wx}{Py} < \frac{1}{2}$$

or $$\frac{W}{P} \cdot \frac{1}{(y/x)} < \frac{1}{2}$$

$$\frac{MA}{VR} < \frac{1}{2}$$

$$\therefore \quad \eta < \frac{1}{2} \text{ or } 50\% \qquad (5.12)$$

Thus, the condition for self-locking or irreversibility of a machine is that the efficiency of machine will always be less than 50%.

*Reversible and Irreversible Machine*

## Condition for Reversibility of Machine

From the definition of a reversible machine it is clear that the machine works in reverse direction only when the output of the machine is greater than the frictional work.

Thus, output > work lost in friction

i.e., $Wx = Py - Wx$

or $2Wx > Py$

$$\frac{Wx}{Py} > \frac{1}{2}$$

or $$\frac{W}{P} \cdot \frac{1}{(y/x)} > \frac{1}{2}$$

$$\frac{MA}{VR} > \frac{1}{2}$$

$\therefore$ $\eta > 1/2$ or 50%

Hence, a machine is said to be reversible if its efficiency is greater than 50%.

# *Law of Machine*

The law of machine gives the relationship between the effort applied and the load lifted. It is obtained by plotting a graph between various efforts required to raise the corresponding loads.

Since $P$ varies linearly with $W$, thus the relationship can be presented mathematically by an equation of a straight line.

$$\therefore \qquad P = mW + C \qquad (5.14)$$

Equation (5.14) is known as law of machine.

where $P$ = Effort applied on machine, $W$ = Load lifted by machine, $C$ = Intercept of the line on $P$-axis, and $m$ = Slope of straight line.

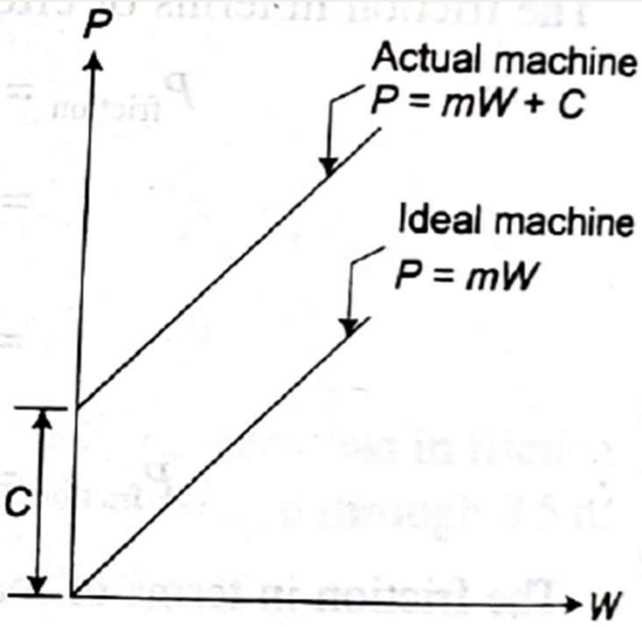



**Fig. 5.1** Variation of Effort against Load

For an actual machine, the graph does not pass through the origin but makes an intercept on the $P$-axis which shows the effort lost in overcoming the friction (Figure 5.1). But for an ideal machine, the line passes through the origin $O$, means $C = 0$, as there is no frictional force.

# *Simple Wheel and Axle*

*Mathmatical Relations*

$$VR = \frac{\pi D}{\pi d}\ Velocity\ Ratio$$

$$MA = \frac{W}{P}\ Mechanical\ Advantage$$

$$\eta = \frac{MA}{VR}\ Efficiency$$

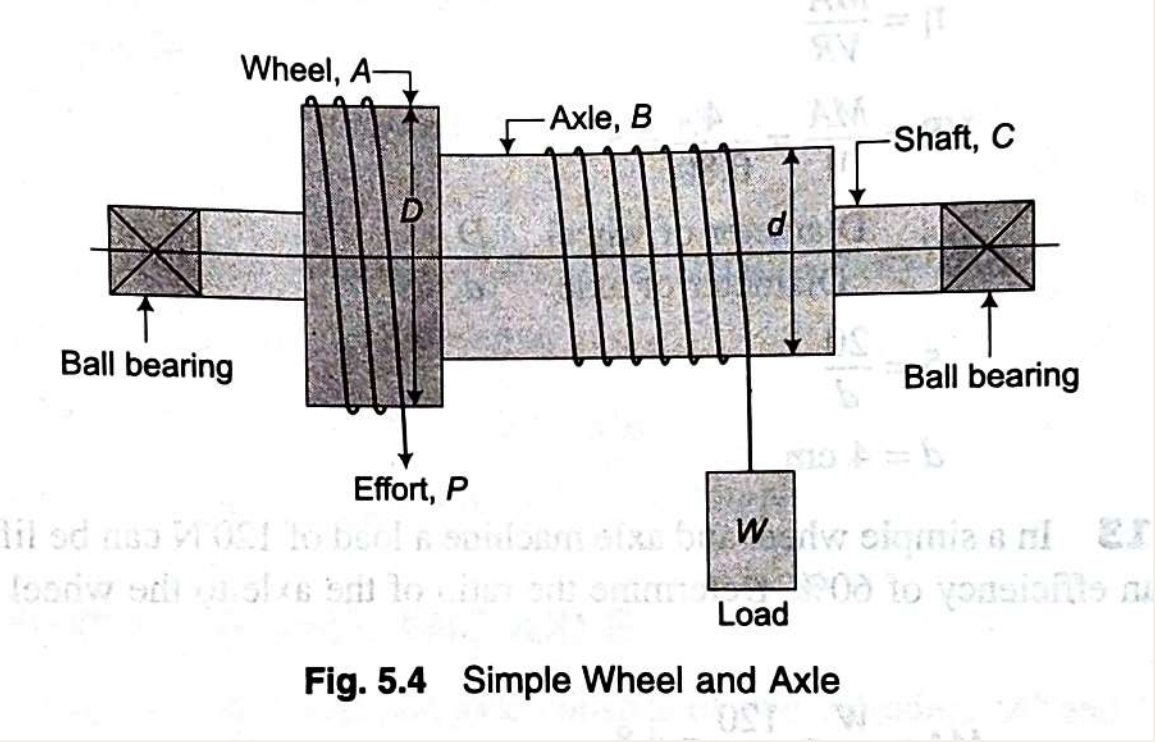



**Fig. 5.4** Simple Wheel and Axle

# Winch

Winch is a machine consisting of a drum on an axle, a pawl, and a crank handle, with or without gearing, to give increased mechanical advantage when hauling on a rope

# Winch Crab

Winch crabs are lifting machines in which the velocity ratio is increased by a gear system.

# *Single Purchase Winch Crabs*

Let $P$ = Effort applied, $W$ = Load lifted, $T_1$ = Number of teeth on the pinion, $T_2$ = Number of teeth on spur gear, $l$ = Length of the lever arm, and $d$ = Diameter of load drum.

In one revolution of the lever, the distance moved by the effort is $2\pi l$.

The pinion also makes one revolution so, the number of revolutions made by the spur gear and the load axle is $T_1/T_2$

$\therefore$ Distance moved by load = $\pi d \times (T_1/T_2)$

$$VR = \frac{\text{Distance moved by effort}}{\text{Distance moved by load}}$$

$$= \frac{2\pi l}{\pi d \times (T_1/T_2)} = \frac{2l}{d} \times \frac{T_2}{T_1} \qquad (5.23)$$

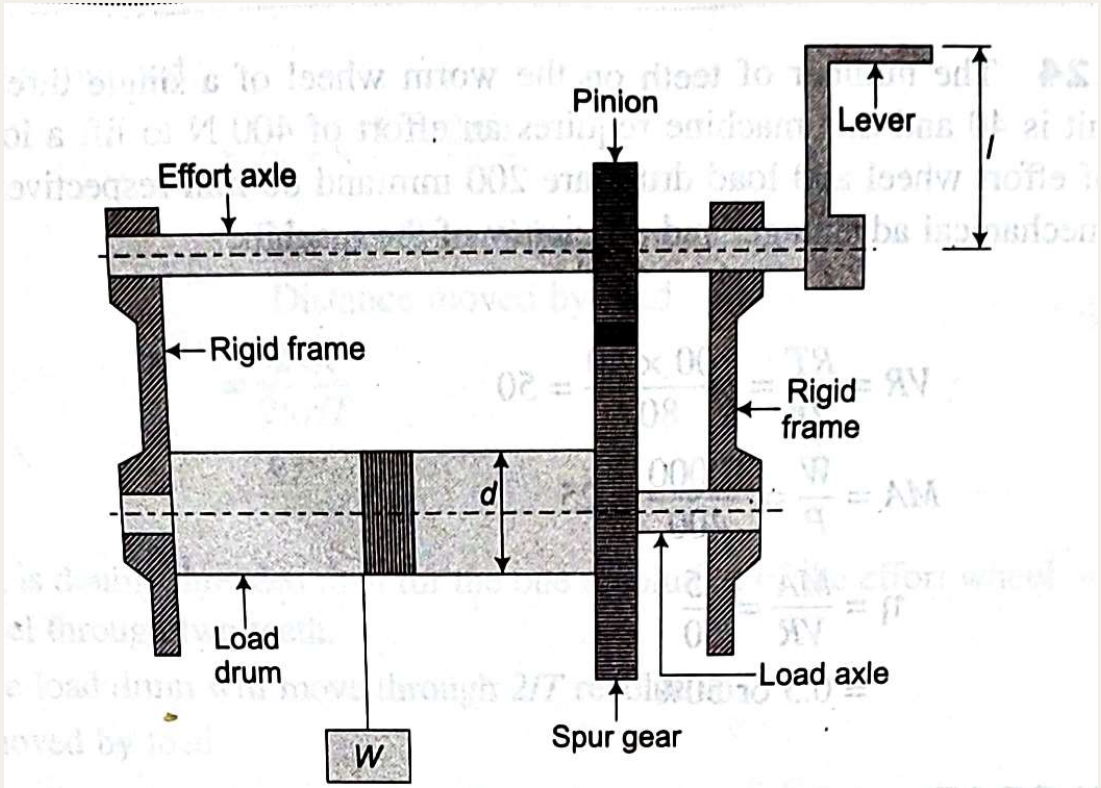



**Fig. 5.8** Single Purchase Winch Crab

$$MA = \frac{\text{Load lifted}}{\text{Effort applied}} = \frac{W}{P}$$

$$\eta = \frac{MA}{VR}$$

# Double Purchase Winch Crabs

Let $P$ = Effort applied, $W$ = Load lifted, $l$ = Length of the lever arm, $d$ = Diameter of the load drum, $T_1$ ≡ Number of teeth on the pinion of the effort axle, $T_2$ = Number of teeth on spur gear of intermediate axle, $T_3$ = Number of teeth on the pinion of intermediate axle, and $T_4$ = Number of teeth on the spur gear of the load axle.

Now for one revolution of the lever,

The distance moved by the effort is $2\pi l$

Number of revolutions made by the pinion of effort axle = 1

Number of revolutions made by spur wheel and pinion of intermediate axle = $1 \times T_1/T_2$

Number of revolutions made by spur wheel and load drum mounted on the load axle

$$= 1 \times \frac{T_1}{T_2} \times \frac{T_3}{T_4}$$

$$\therefore \text{ Distance moved by load} = \pi d \times \left(1 \times \frac{T_1}{T_2} \times \frac{T_3}{T_4}\right)$$

$$= \pi d \times \frac{T_1}{T_2} \times \frac{T_3}{T_4}$$

$$VR = \frac{\text{Distance moved by effort}}{\text{Distance moved by load}}$$

$$= \frac{2\pi l}{\pi d \times (T_1/T_2) \times (T_3/T_4)}$$

$$= \frac{2l}{d} \times \frac{T_2}{T_1} \times \frac{T_4}{T_3}$$

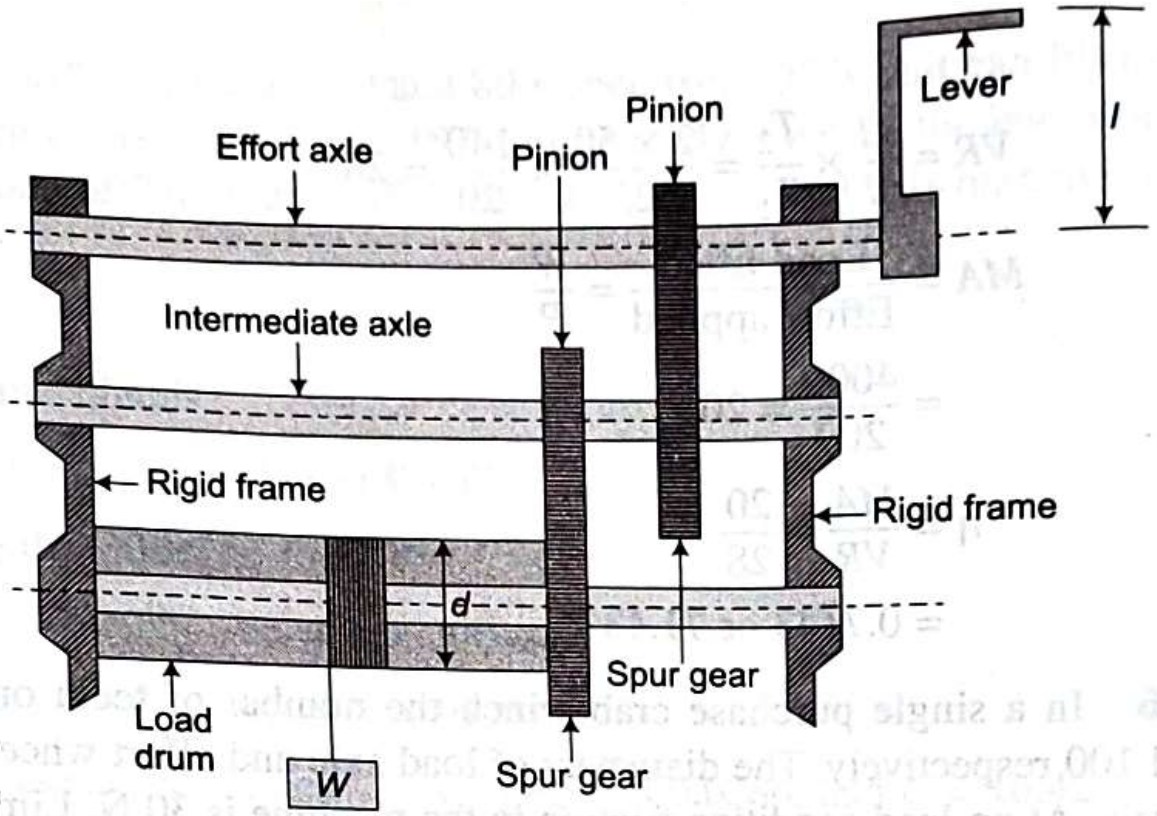
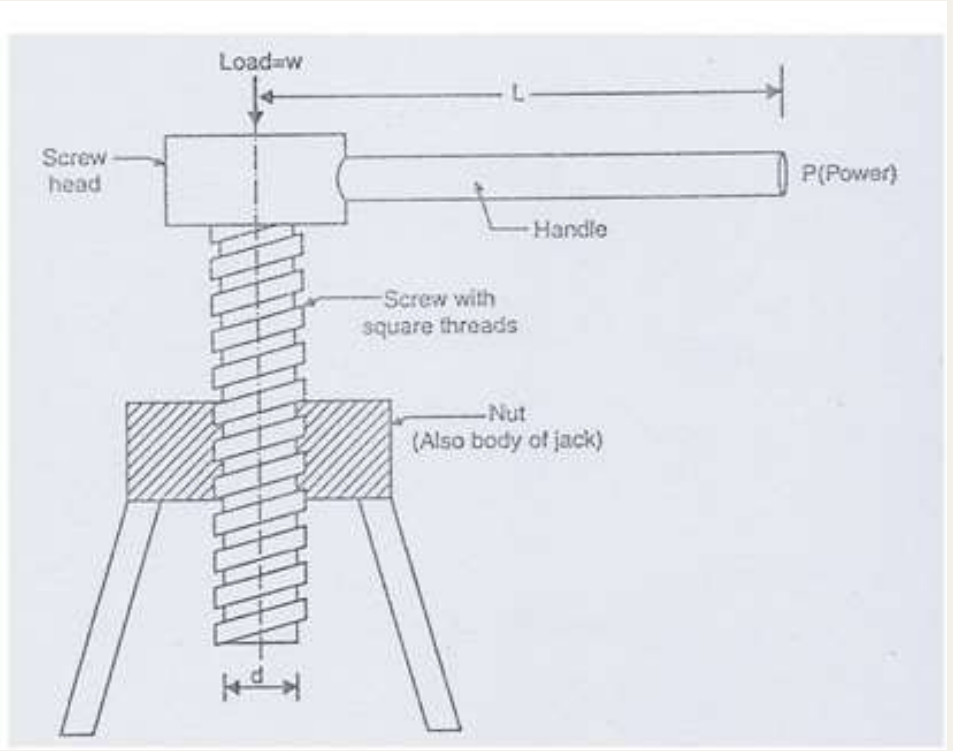


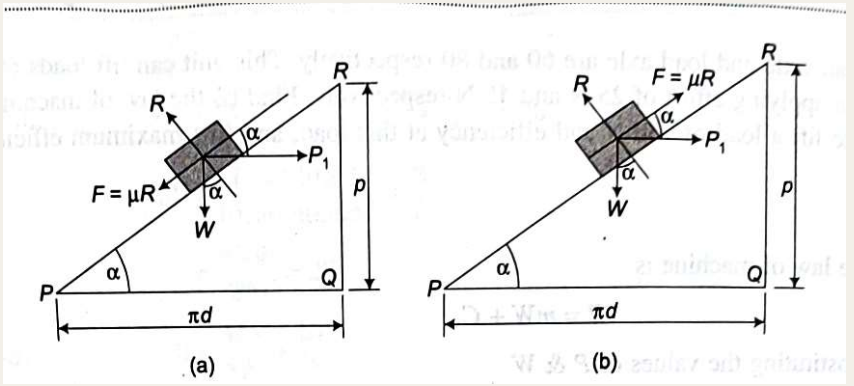

**Fig. 5.9** Double Purchase Winch Crab

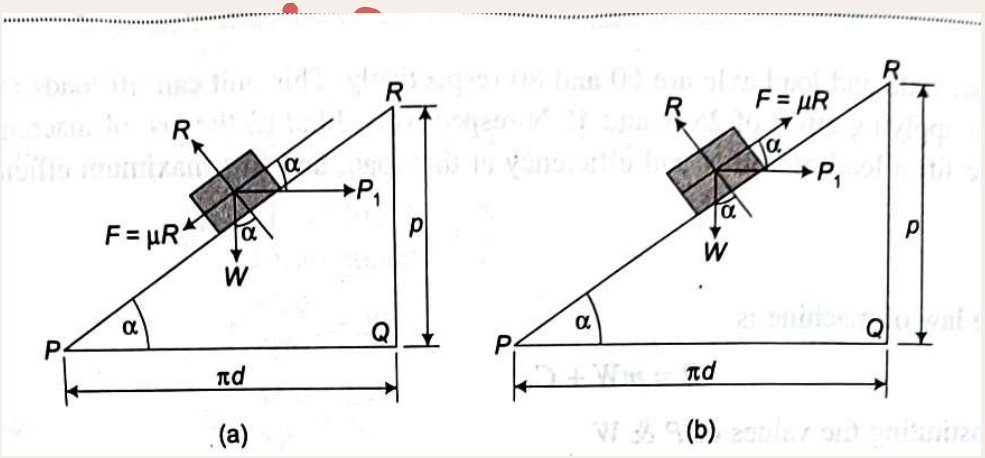
$$MA = \frac{\text{Load lifted}}{\text{Effort applied}} = \frac{W}{P}$$

$$\eta = \frac{MA}{VR}$$

# *Screw Jack*


Load=w
L
Screw head
P(Power)
Handle
Screw with square threads
Nut
(Also body of jack)
d


R
α
P1
F = μR
W
p
P
Q
πd
(a)
(b)




### 5.10.1 Simple Screw Jack

A simple screw jack consists of a square threaded screw which moves inside a heavy nut which forms the base of the unit. A handle is fitted to the screw head. The load to be lifted is placed on the head of screw which is lifted or lowered by the application of an effort at the end of the handle as shown in Figure 5.11.

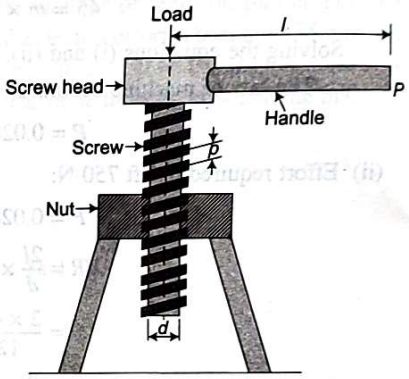



**Fig. 5.11 Simple Screw Jack**

Let $d$ = Mean diameter of the screw = Average diameter of the root diameter and outer diameter of the screw, $p$ = Pitch of the screw = Distance between two successive threads, $\alpha$ = Helix angle or screw angle, $\phi$ = Angle of friction, $l$ = Length of the handle, $W$ = Load lifted, $P$ = Effort applied at the end of the handle, and

$$\mu = \tan \phi = \text{Coefficient of friction}$$

In one revolution of the screw the load is moved by a distance equal to the pitch of the screw.

$\therefore$ Distance moved by load = $p$

Distance moved by effort = $2\pi l$

Velocity ratio, $$VR = \frac{\text{Distance moved by effort}}{\text{Distance moved by load}}$$

$$= \frac{2\pi l}{p} \qquad (5.25)$$

If the screw is double threaded then distance moved by load = $2p$

Therefore, $$VR = \frac{2\pi l}{2p} = \frac{\pi l}{p} \qquad (5.26)$$

$$MA = \frac{\text{Load lifted}}{\text{Effort applied}} = \frac{W}{P}$$

$$\eta = \frac{MA}{VR}$$

## Expression for Effort Required to Lift the Load

When the handle completes one revolution, the screw is also rotated through one turn and load is lifted by a height equal to pitch of the screw. The development of one complete revolution of a screw thread is shown in Figure 5.10 (a). The distance $PQ$ will be equal to the circumference ($\pi d$) and the distance $QR$ will be equal to the pitch ($p$) of the screw. From the geometry of the Figure (5.10 a), we have,

$$\tan \alpha = \frac{QR}{PQ} = \frac{p}{\pi d} \qquad (5.27)$$

Let $P_1$ is the effort applied horizontally at the mean radius of the screw jack to lift the load $W$, $R$ is the normal reaction, and $F$ is the force of friction = $\mu R$

When the load $W$ is lifted upwards, the force of friction $F$ acts downwards.

Resolving the forces along the inclined plane,

$$F + W \sin \alpha = P_1 \cos \alpha$$

or $$\mu R + W \sin \alpha = P_1 \cos \alpha \quad (\because F = \mu R) \qquad (i)$$

Resolving the forces normal to the inclined plane,

$$R = W \cos \alpha + P_1 \sin \alpha \qquad (ii)$$

Substituting equation (ii) in (i),

$$\mu (W \cos \alpha + P_1 \sin \alpha) + W \sin \alpha = P_1 \cos \alpha$$

But $$\mu = \tan \phi = \frac{\sin \phi}{\cos \phi}$$

$\therefore$ $$\frac{\sin \phi}{\cos \phi} (W \cos \alpha + P_1 \sin \alpha) + W \sin \alpha = P_1 \cos \alpha$$

or $$W \frac{\sin \phi \cos \alpha}{\cos \phi} + P_1 \frac{\sin \phi \sin \alpha}{\cos \phi} + W \sin \alpha = P_1 \cos \alpha$$

Multiplying by $\cos \phi$,

$$W \sin \phi \cos \alpha + P_1 \sin \phi \sin \alpha + W \sin \alpha \cos \phi = P_1 \cos \alpha \cos \phi$$

$$W [\sin \phi \cos \alpha + \sin \alpha \cos \phi] = P_1 [\cos \alpha \cos \phi - \sin \phi \sin \alpha]$$

or $$W \sin (\alpha + \phi) = P_1 \cos (\alpha + \phi)$$

$\therefore$ $$P_1 = W \frac{\sin (\alpha + \phi)}{\cos (\alpha + \phi)}$$

$$= W \tan (\alpha + \phi) \qquad (5.28)$$

$P_1$ is the effort applied at the mean radius of the screw jack but actually the effort is applied at the end of the handle as shown in Figure 5.11.

Moment of $P_1$ about the axis of the screw = $P_1 \times (d/2)$

Moment of $P$ about the axis of the screw = $P \times l$

Equating the two moments, we have,

$$P_1 \times \frac{d}{2} = P \times l$$

or $$P_1 = P \times \frac{2l}{d}$$

Substituting the value of $P_1$ in Eq. (5.28), we have,

$$P \times \frac{2l}{d} = W \tan (\alpha + \phi)$$

$\therefore$ $$P = \frac{d}{2l} \times W \tan (\alpha + \phi) \qquad (5.29)$$

Equation (5.29) gives the relation between the effort required and the load lifted.

The value of $P$ in terms of pitch $p$ and coefficient of friction $\mu$:

We know that $$P = \frac{d}{2l} \times W \tan (\alpha + \phi)$$

$$= \frac{Wd}{2l} \times \frac{\tan \alpha + \tan \phi}{1 - \tan \alpha \tan \phi}$$

or $$P = \frac{Wd}{2l} \times \frac{(p/\pi d) + \mu}{1 - (p/\pi d) \cdot \mu}$$

$\therefore$ $$P = \frac{Wd}{2l} \left( \frac{p + \mu \pi d}{\pi d - p\mu} \right) \qquad (5.30)$$

Equation (5.30) gives the value of $P$ in terms of $\mu$ and $p$.

## Expression for Effort Required to Lower the Load

When the load is lowered by the screw jack, the force of friction $F$ will act upwards as shown in Figure 5.10 (b).

Resolving the forces along the inclined plane,

$$F + P_1 \cos \alpha = W \sin \alpha$$

or $$\mu R + P_1 \cos \alpha = W \sin \alpha \qquad (i)$$

Resolving the forces normal to the inclined plane,

$$R = W \cos \alpha + P_1 \sin \alpha$$

Substituting the values of $R$ in equation (i),

$$\mu (W \cos \alpha + P_1 \sin \alpha) + P_1 \cos \alpha = W \sin \alpha$$

$$\mu W \cos \alpha + \mu P_1 \sin \alpha + P_1 \cos \alpha = W \sin \alpha$$

or $$\mu P_1 \sin \alpha + P_1 \cos \alpha = W \sin \alpha - \mu W \cos \alpha$$

$$P_1 (\mu \sin \alpha + \cos \alpha) = W (\sin \alpha - \mu \cos \alpha)$$

But $$\mu = \tan \phi = \frac{\sin \phi}{\cos \phi}$$

$$\therefore \quad P_1 \left( \frac{\sin \phi}{\cos \phi} \sin \alpha + \cos \alpha \right) = W \left( \sin \alpha - \frac{\sin \phi}{\cos \phi} \cos \alpha \right)$$

Multiplying by $\cos \phi$, we get,

$$P_1 (\sin \phi \sin \alpha + \cos \alpha \cos \phi) = W (\sin \alpha \cos \phi - \sin \phi \cos \alpha)$$

or $$P_1 \cos (\phi - \alpha) = W \sin (\phi - \alpha)$$

$$\therefore \quad P_1 = W \frac{\sin (\phi - \alpha)}{\cos (\phi - \alpha)}$$

$$= W \tan (\phi - \alpha) \qquad (5.31)$$

If $\alpha > \phi$ then

$$P_1 = W \tan (\alpha - \phi) \qquad (5.32)$$

$P_1$ is the effort applied at the mean radius of the screw jack but in actual practice the effort $P$ is applied at the end of the handle of the jack. When the effort $P$ is applied at the end of the handle as shown in Figure 5.11, then equating the moments of $P$ and $P_1$ about the axis of the jack, we get,

$$P \times l = P_1 \times \frac{d}{2}$$

or $$P = \frac{d}{2l} \times P_1$$

Now substituting the value of $P_1$ from equation (5.31), we have,

$$P = \frac{d}{2l} \times W \tan (\phi - \alpha) \qquad (5.33)$$

Equation (5.33) gives the relation between effort required at the end of the handle to lower the load.

The value of $P$ in terms of pitch $p$ and coefficient of friction $\mu$:

We know that

$$P = \frac{d}{2l} \times W \tan (\phi - \alpha)$$

$$= \frac{Wd}{2l} \times \frac{\tan \phi - \tan \alpha}{1 + \tan \phi \tan \alpha}$$

$$= \frac{Wd}{2l} \times \frac{\mu - (p/\pi d)}{1 + \mu (p/\pi d)}$$

$$\therefore \quad P = \frac{Wd}{2l} \left( \frac{\mu \pi d - p}{\pi d + \mu p} \right) \qquad (5.34)$$

## Expression for Efficiency of a Screw Jack

The actual effort $P$ required to lift the load $W$ is given by equation (5.29),

$$P = \frac{d}{2l} \times W \tan (\alpha + \phi) \qquad (i)$$

In the absence of friction $\phi$ will become zero. In that case, effort $P$ will become ideal effort $P_i$,

$$\therefore \quad P_i = \frac{d}{2l} \times W \tan \alpha \qquad (ii)$$

$$\eta = \frac{\text{Ideal effort}}{\text{Actual effort}}$$

$$= \frac{P_i}{P_a} = \frac{(d/2l) \times W \tan \alpha}{(d/2l) \times W \tan (\alpha + \phi)}$$

$$\therefore \quad \eta = \frac{\tan \alpha}{\tan (\alpha + \phi)} \qquad (5.35)$$

## Expression for Maximum Efficiency

The condition for maximum efficiency can be obtained by,

$$\frac{d\eta}{d\alpha} = 0$$

or $$\frac{d}{d\alpha} \left[ \frac{\tan \alpha}{\tan (\alpha + \phi)} \right] = 0$$

$$\frac{\tan (\alpha + \phi) \sec^2 \alpha - \tan \alpha \sec^2 (\alpha + \phi)}{\tan^2 (\alpha + \phi)} = 0$$

or $$\tan (\alpha + \phi) \sec^2 \alpha - \tan \alpha \sec^2 (\alpha + \phi) = 0$$

$$\frac{\sin (\alpha + \phi)}{\cos (\alpha + \phi)} \times \frac{1}{\cos^2 \alpha} = \frac{\sin \alpha}{\cos \alpha} \times \frac{1}{\cos^2 (\alpha + \phi)}$$

or $$\sin (\alpha + \phi) \cos (\alpha + \phi) = \sin \alpha \cos \alpha$$

$$\sin 2(\alpha + \phi) = \sin 2\alpha = \sin (\pi - 2\alpha)$$

or $$2(\alpha + \phi) = (\pi - 2\alpha)$$

or
$$4\alpha = \pi - 2\phi$$
$$\alpha = \frac{\pi}{4} - \frac{\phi}{2} = 45° - \frac{\phi}{2} \quad (5.36)$$

Hence, maximum efficiency can be obtained by substituting the value of $\alpha$ in equation (5.35), we get,

$$\eta_{max} = \frac{\tan(45° - \phi/2)}{\tan(45° + \phi/2)} = \frac{\dfrac{1 - \tan\phi/2}{1 + \tan\phi/2}}{\dfrac{1 + \tan\phi/2}{1 - \tan\phi/2}}$$

$$= \frac{(1 - \tan\phi/2)^2}{(1 + \tan\phi/2)^2} = \frac{\left(\dfrac{\cos\phi/2 - \sin\phi/2}{\cos\phi/2}\right)^2}{\left(\dfrac{\cos\phi/2 + \sin\phi/2}{\cos\phi/2}\right)^2}$$

$$= \frac{(\cos\phi/2 - \sin\phi/2)^2}{(\cos\phi/2 + \sin\phi/2)^2}$$

$$= \frac{\cos^2\frac{\phi}{2} + \sin^2\frac{\phi}{2} - 2\cos\frac{\phi}{2}\sin\frac{\phi}{2}}{\cos^2\frac{\phi}{2} + \sin^2\frac{\phi}{2} + 2\cos\frac{\phi}{2}\sin\frac{\phi}{2}}$$

$$\therefore \quad \eta_{max} = \frac{1 - \sin\phi}{1 + \sin\phi} \quad \left(\because \cos^2\frac{\phi}{2} + \sin^2\frac{\phi}{2} = 1,\ 2\cos\frac{\phi}{2}\sin\frac{\phi}{2} = \sin\phi\right) \quad (5.37)$$

### 5.10.2 DIFFERENTIAL SCREW JACK

A differential screw jack is shown Figure 5.12.

It consists of two threaded parts say '*A*' and '*B*'. Part '*A*' has only external threads while part '*B*' has both internal as well as external threads. The threads of screw '*A*' engage inside the internal threads of screw '*B*', while the external threads of screw '*B*' engage inside the threads of the nut '*C*' which forms the main body of this unit. So, part '*B*' acts as a nut for screw *A* and acts as a screw for nut '*C*'.

Let $W$ = Load lifted, $P$ = Effort applied, $l$ = Length of the handle, $p_a$ = Pitch of screw *A*, and $p_b$ = Pitch of screw *B*

Let the handle complete one revolution,

Distance moved by effort = $2\pi l$

Distance moved by screw *B* in upward direction = $p_b$

Distance moved by screw *A* in downward direction = $p_a$

$\therefore$ The net distance through which load is lifted = $p_b - p_a$

$$VR = \frac{\text{Distance moved by effort}}{\text{Distance moved by load}}$$

$$= \frac{2\pi l}{p_b - p_a} \quad (5.38)$$

$$MA = \frac{\text{Load lifted}}{\text{Effort applied}} = \frac{W}{P}$$

and
$$\eta = \frac{MA}{VR}$$

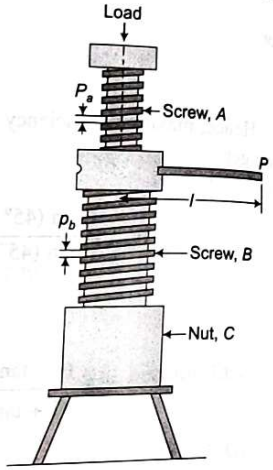



**Fig. 5.12** Differential Screw Jack

**Example 5.28** To lift a load of 3000 N an effort of 75 N is applied at the end of a handle of length 150 mm. Determine the efficiency of the single screw jack if the pitch of the screw is 10 mm.

***Solution***

W = 3000